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대류가 우세한 상변화 문제 해석을 위한 고정격자 유한체적법

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A Fixed-Grid Finite Volume Approach to Convection-Dominated Phase Change Problems

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ABSTRACT

A fixed-grid finite volume numerical approach is developed to simulate physical details of convection-dominated phase change problems. This approach adopts the enthalpy-porosity method associated with new algorithms that is devised to track the phase front efficiently. A comparative analysis with transformed- and fixed-grid approaches is performed to demonstrate the predictability of the presented model. Results of a melting and solidification experiments are used to assess and evaluate the performance of the model.

Key Words : phase change problem, fixed-grid approach, finite volume method, convection

I. INTRODUCTION

The analysis on the melting and solidification has received a great attention in last several decades.[1] The natural convection in the melt due to the temperature difference can significantly affect the phase change process and then the morphology of the solid-liquid interface is influenced by changing the flow structure in the melt.

This study attempts a numerical analysis on convection-dominated phase change problem with the fixed-grid method, which uses a single set of conservation equations and boundary conditions for the whole domain comprising the solid and liquid phases. In this method, the interface conditions are described as suitable source terms in the governing equations to avoid explicit treatment of the interface. Upon phase changing, the latent heat absorption, or evolution, is reflected as a source, or sink, term in the energy equation. Also, the fixed-grid requires the velocity suppression because as a liquid region turns solid, the zero-velocity condition should be satisfied. The velocity suppression is accomplished by a suitable source term in the momentum equation to model the two-phase domain as a porous medium.[2] The fixed-grid combined with the porous medium method is usually referred to as the enthalpy-porosity

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method.

To simulate the phase change process efficiently, this paper proposes a simple numerical algorithm based on the prediction-correction of the temperature and the liquid fraction of the cells undergoing phase change.

II. MATHEMATICAL MODELS

The flow is assumed to be two-dimensional, laminar, and incompressible. The physical properties of the material are constant in each phase. The density difference between solid and liquid phases is negligible except when utilizing the Boussinesq approximation.

The enthalpy formulation defines the liquid mass fraction f as the ratio of the liquid mass to the total mass in a given computational cell. If the saturation enthalpy of solid phase h_s and the melting temperature T_m are set to the reference enthalpy and temperature, respectively, the specific enthalpy will be

$$h = fL + cT \tag{1}$$

In this, L is the latent heat. The heat capacity c may vary with the phase. The liquid mass fraction can be obtained from the enthalpy:

$$f = \begin{cases} 0 & if\ h < 0 \\ \dfrac{h}{L} & if\ 0 \le h \le L \\ 1 & if\ L < h \end{cases} \tag{2}$$

For the velocity suppression, this study introduces a Darcy-like momentum source term, which has functional form of[2]

$$-C\frac{(1-f)^2}{(f^3+b)}u^p \tag{3}$$

The constant C has a large value to suppress the velocity as the cell becomes solid and b is a small number used to prevent the division-by-zero when a cell is fully located in the solid region, namely $f=0$. In this work, $C=1\times10^9 kg/m^3 s$ and $b=0.005$ are used.[2]

III. NUMERICAL METHODS

The SIMPLE algorithm[3] is employed to determine the velocity and pressure field. The deferred correction method[3] is introduced for the interpolation scheme of the convection term. As the lower - and the higher-order scheme, the upwind difference scheme and the central difference scheme are chosen, respectively. The case that both schemes equally contribute to the convection term is referred as the mixed difference scheme.

The discretized energy equation in the finite volume formulation[3] can be expressed as

$$a_P T_P = \sum_{nb} a_{nb} T_{nb} + S_P - A_P^0 (f_P - f_P^*) \tag{4}$$

where subscripts 'P' and 'nb' refer to the value of present and neighboring cell, respectively. Superscript '*' denotes the value at previous time step. With the temperature field obtained at the n-th iteration step, the enthalpy and the liquid fraction can be calculated from Eqs (1) and (2), respectively. This procedure, which is the main idea of the enthalpy method, enables the energy contained in the cell to redistribute so that the excessive(or deficient) energy can be stored into(or retrieved from) latent heat rather than spurious sensible heat.

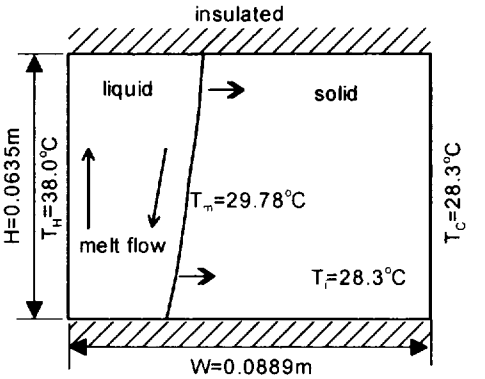
To improve the computational procedure, consider the phase changing cells. If we assume the present cell 'P' is undergoing phase change, the temperature is given by

$$T_P^{(n+1)} = \frac{\sum_{nb} a_{nb}^{(n+1)} T_{nb}^{(n)} GS_P^{(n+1)}}{a_P^{(n+1)}} \quad (5)$$

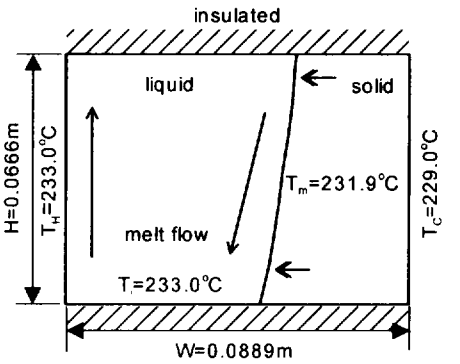
The generalized source GS_P can be easily obtained from Eq(4). The influence coefficients and the generalized source are calculated with the updated mass fraction $f^{(n+1)}$. Although the temperatures of neighboring cells T_{nb} are obtained at the previous iteration step, the new temperature $T_P^{(n+1)}$ based on the updated mass fraction will be a more accurate estimation. This temperature can be used to update the mass fraction with the enthalpy expression. This predictor-corrector procedure will be applied iteratively only to phase changing cells consisting in the phase change front. The whole set of the governing equations do not need to be solved during this procedure. The proposed simple but effective algorithm always ensures the energy conservation at the phase changing cell. Furthermore, this can be readily adaptable to any numerical scheme designed for computational efficiency.

IV. NUMERICAL RESULTS

The convection-dominated melting of a pure gallium[4] is simulated with the proposed algorithm. The present numerical results are compared with the transformed-grid results[2] and the finite element solutions.[5] The experimental configuration is sketched in Figure 1.



(a) Melting of gallium



(b) Solidification of tin

Fig. 1. Schematics of phase change problems in a rectangular cavity

The predicted phase change fronts are shown with the results from the experiment and other numerical models in Figure 2. The convection term is interpolated based on the mixed difference scheme. When analyzing solid-liquid interfaces at 6 and 10 min, the finite element and the transformed -grid solutions seem to give better prediction than the present calculation. However, even though the hot wall temperature is assumed to reach the desired temperature upon starting the experiment, to raise the temperature impulsively to the desired one is very difficult in practice. The actual amount of energy transferred to the gallium through the hot wall should be less than that imposed in the idealized calculation, so that the retardation of the

front evolution in the experiment may be likely. At 19 min when the effect of delayed heat-up at the hot wall is less significant, the interfaces obtained with the fixed-grid correspond better than those with the transformed-grid do.

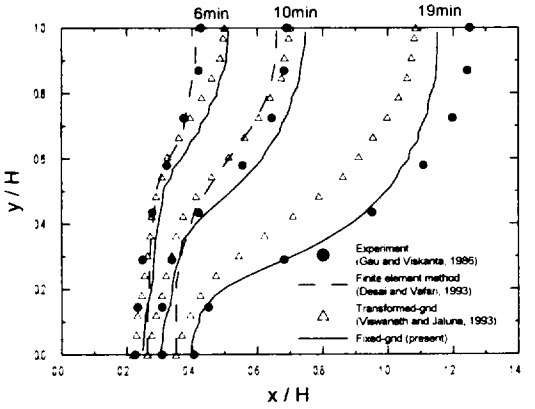


Fig. 2. Phase change fronts during the gallium melting

In order to show the flow structure in the melt, the streamlines are plotted in Figure 3. An interesting point to note is that the flow structures obtained in previous studies[1] show only a single cell in the melt region. With the transformed-grid, Viswanath and Jaluria[2] observed a secondary recirculation cell in the lower part of the melt region. They used the power scheme for the interpolation of the convection term. In the present study, the mixed difference scheme shows the obvious distortion of the streamlines in the lower melt region. On the other hand, although not shown here, the streamlines by the upwind difference and the power schemes fail to predict the minor flow motion in the melt.

The solidification of tin is also simulated with the proposed model. The experimental and the transformed-grid results by Wolff and Viskanta[6] shown in Figure 1(b) are cited for the comparison.

Figure 4 shows the temperature distribution in the liquid during the solidification of tin. The present fixed-grid solutions are very similar to the transformed-grid results. Considering some of the scatter in the measured temperatures as noted by Wolff and Viskanta.[6] The results generated from numerical methods are in fairly good agreement with the experimental data.

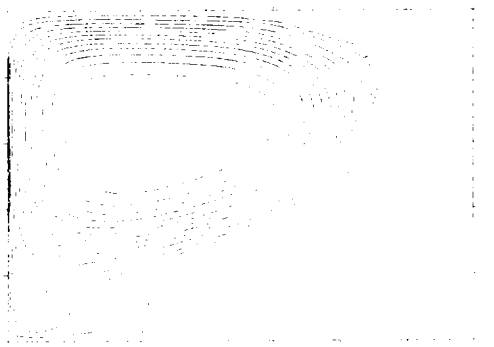


Fig. 3. Streamlines for gallium melting at 19min

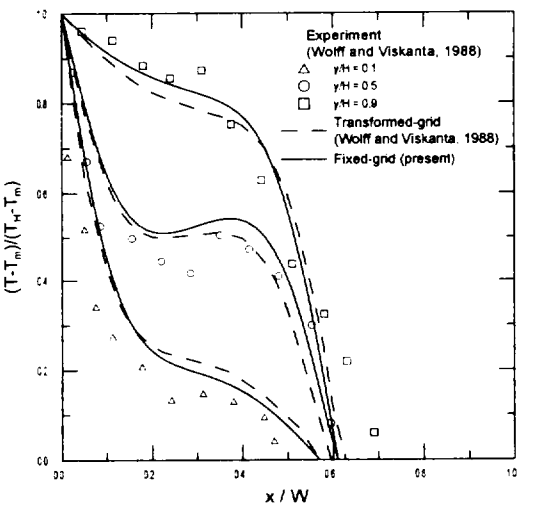


Fig. 4. Temperature profile in the melt during the solidification of tin at 0.529 hr

V. CONCLUSIONS

The convection-dominated melting and solidification in a rectangular cavity is investigated numer-

ically on the basis of the fixed-grid formulation. The phase change process and the velocity suppression is modeled by the enthalpy-porosity method.

The gallium melting and the tin solidification are simulated with the present model and the results are comparable with the experimental data in the literature. The comparison with the transformed-grid calculation shows that the present model produces similar or better predictions of the macroscopic feature of the melting like the movement of the phase change front.

요 약

대류가 우세한 상변화 문제의 물리적 거동을 모사하기 위해서 고정격자 유한체적법을 도입하였다. 이 연구에서는 엔탈피-다공성 방법을 채택하였으며, 상변화 경계면을 정확히 추적하기 위해 고안된 새로운 알고리즘이 함께 사용하였다. 제안된 모델의 예측 능력을 평가하기 위해 변환격자법 및 고정격자법에 의해 계산된 결과와 비교하였다. 그리고 용융 및 응고 실험의 결과와도 비교하였다.

참고문헌

1) V. R. Voller. 1997. An Overview of Numerical Methods for Solving Phase Change Problems. In: Minkowycz WJ, Sparrow EM, editors. Advanced Numerical Heat Transfer I. Washington DC: Taylor & Francis.

2) R. Viswanath and Y. Jaluria, 1993. A Comparison of Different Solution Methodologies for Melting and Solidification Problems, Numerical Heat Transfer Part, Vol 24, pp. 77~105.

3) J. H. Ferziger and M. Peric, 1999. Computational Methods for Fluid Dynamics, Springer, Germany.

4) 'C. Gau and R. Viskanta, 1986. Melting and Solidification of a Pure Metal on a Vertical Wall, J. Heat Transfer, Vol. 108, pp. 174~181.

5) C. P. Desai and K. Vafai, 1993. A Unified Examination of the Melting Process within a Two-Dimensional Rectangular Cavity. J, Heat Transfer, Vol. 115, pp. 1072~1075.

6) F. Wolff and R. Viskanta, 1988. Solidification of a Pure Metal at a Vertical Wall in the Presence of Liquid Superheat. Int. J. Heat Mass Transfer, Vol. 31, pp. 1735~1744