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A New Correction Algorithm of Servo Track Writing Error for High Track Density Disk Drives

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국문초록

디스크 드라이브는 현대 컴퓨터 시스템과 네트워크의 1 차 저장장치로서 사용되고 있다. 전형적인 디스크 드라이브는 변화없는 높은 속도로 회전하는 스핀들 모터에 장착된 하나 혹은 여러 개의 강 마그네틱 저장 디스크를 포함하고 있다. 하드 디스크 드라이브의 개발 경향은 더 작고 트랙 폭이 좁으며 더 많은 용량을 가지는 쪽으로 연구하고 있다. 특히 헤드의 정확한 위치제어는 디스크 드라이브에서 더 높은 트랙 밀도의 증가를 위한 주 선행과제이다. 이러한 문제를 해결하기 위해 일반적으로 사용된 서보 제어기는 외란이 존재하더라도 헤드를 트랙의 정 중앙에 위치시키는 것이다. 하드 디스크 드라이브의 서보 트랙은 서보 트랙 기록기 (servo track writer)라 불리는 장치의 도움을 받아 생산과정에서 기록을 한다. 서보트랙 기록 과정에서의 디스크 혹은 헤드의 진동은 서보트랙 기록오차 (servo track writing error)를 발생시킨다. 트랙의 교란된 형태는 서보 제어기가 회전하는 디스크의 중심에 대한 트랙 중심선의 반지름이 연속적으로 변화하는 것을 따라가기 위하여 트랙 추종을 하는 동안 헤드의 위치를 재 선정 해야 하므로 헤드 위치선정을 복잡하게 만든다. 그리고 서보 제어기는 저장시 외부 진동에 의한 헤드진동으로 외부트랙과 내부트랙의 원치않는 저장을 일으켜

이로 인한 데이터 손실이 발생되며, 발생된 데이터의 복구는 불가능하다. 특히, 디스크 드라이브의 수행에 직접적으로 영향을 미치는 고밀도 트랙 디스크 드라이브의 경우는 더욱 심각하다. 따라서 트랙 피치의 제한을 피하고 디스크 드라이브의 신뢰성 있는 작업을 위해서는 서보트랙 기록오차를 예측하고 그것을 보상해야한다.

본 논문에서는 서보트랙 기록오차를 제거하기 위한 새로운 방법을 제안하였으며, 제안된 방법은 서보트랙 기록오차를 예측하기 위하여 위치 오차 신호와 제어기 출력값을 사용하였고 피드포워드(feed forward manner) 방식으로 오차를 예측하여 보상하였다. 본 논문에서 제안한 알고리즘은 서보트랙 기록오차 감소에 매우 효과적이며, 외란에 견딜 수 있는 강인함과 이전의 알고리즘에 비해 빠른 수렴율을 갖는다. 마지막으로 서보트랙 기록오차에서 수정된 알고리즘의 유효성을 증명하는 시뮬레이션 결과를 보였다.

SUMMARY

Disk drives serve as the primary storage medium in modern computer systems and networks. A typical disk drive comprises of one or more rigid magnetic storage discs mounted on the spindle motor which rotates at a constant high speed. The prevalent trend of hard disk drives is toward small hard disks with increasing larger capacities which implies track width has to be small. Accurate positioning of head is one of the main prerequisite for the increase of further track density in disk drives. The objective of servo controller is to position the head at the center of track in the presence of external disturbances. The servo tracks of hard disk drives are written at the time of manufacture with the help of equipment called *servo track writer*. The vibration of disk or head during servo writing process induces *servo track writing error*. The perturbed shaped of the tracks complicates the head positioning because servo controller needs to reposition the head during the track following to keep up with the constantly changing radius of the track centerline with respect to the center of the spinning disk. This induces unwanted head motion thus causes interference i.e. inner track path gets interfered with the outer track path and hence may cause the customers data to be lost and hence not recoverable data error. Especially, this is severe in case of high track density disk drives which effects directly performance of disk drives.

Therefore, to avoid limitations on the track pitch and for the reliable operation of the disk drive, we need to estimate the *servo track writing error* and compensate for it.

In this study, we propose a new way of correcting for the servo track writing error. We introduce state predictor that uses position error signal and controller output to estimate the *servo track writing error*. The predictor error has lot of information about servo track writing error. Therefore, predictor error is processed to estimate servo track writing error which is compensated in feed forward manner. Our proposed algorithm is robust to input disturbances. Moreover, it is efficient in reducing servo track writing error, computationally simple and has fast convergence compared to previous algorithms. Finally, we present the simulation results to demonstrate that the proposed algorithm is very effective in reducing the servo track writing error.

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I. INTRODUCTION

Hard disk drives serve as an important data storage medium for data processing systems. The data storage density of hard disk drives is continually increasing due to the advent of giant magneto-resistive (GMR) head. The track density of the current hard disk drives is 168,000 tracks per inch (TPI) and the equivalent track pitch is 0.15 μm. Data is read from or written to a track with the help of giant magneto-resistive (GMR) head which converts digital data into magnetic pulses and vice versa. The head floats several micro inches over the disk surface while the disk spins about its center at a constant angular velocity. To properly locate head during read or write process, a closed-loop servo mechanism is generally implemented that uses the servo data written to the disk surface to align the head to the desired track. Accurate positioning of heads is one of the most important prerequisites for extending further the track density in hard disk drives. The mechanical disturbances contributing primarily to the positioning error can be divided into two categories, *repeatable runout* (RRO) and *nonrepeatable runout* (NRRO) (Abramovitch *et al.,* 1998; Guo *et al.,* 1999). The source of *RRO* is the discrepancy between the center of servo tracks and the center of spindle motor, while *NRRO* originates from several sources such as disk flutter, spindle vibration, and windage induced slider vibration.

Much research has been devoted to the compensation of the mechanical disturbances, especially *RRO* disturbance. The periodic nature of *RRO* disturbance allowed the adaptive feed-forward canceller (Messner *et al*., 1994; Sacks *et al.,* 1995; Weerasooriya *et al.,* 1996) or repetitive controller (Kempf *et al.,* 1992; Tomizuka *et al.,* 1989) to reduce significantly the effect of *RRO* disturbance. The repetitive controller can be classified as internal model based and external model based. Internal model based controllers use Q-filter algorithm where as external based controllers use parameters adaptation scheme and learning algorithm. Repetitive controllers developed using *Internal Model Principle* (Kempf *et al.,* 1992; Tomizuka *et al.,* 1989) can provide exact cancellation of the periodic disturbance in steady state but it is not robust to system parameter variations when it is disconnected from the feedback loop. The internal model based approaches converged fast but changed the loop gain i.e. the magnitude of the intermediate harmonics are changed. In case of external model based once converged does not change the loop gain. The adaptive *RRO* compensators (Messner *et al.,* 1994; Sacks *et al.,* 1995; Weerasooriya *et al.,* 1996) are most widely used in industry because of robustness to plant parameter variations and simplicity in implementation. In (Sacks *et al.,* 1995) various adaptive feed forward methods for *RRO* compensation are proposed. Of the adaptive feed forward methods, the feed through technique is superior. Multiple cancellation of harmonics using adaptive feed forward cancellation (AFC) is not effective as the repetitive controller and the convergence rate is slow compared to repetitive controller. AFC can remove the harmonics selectively while repetitive controller removes all integer multiples of harmonics. Apart from the case of *RRO,* the inverse notch filter at the predicted *NRRO* frequency has been used in industry for the attenuation of *NRRO* disturbance.

As the track density becomes higher, another kind of disturbance that is not well known to the academic world has more significant effect on the performance of disk drives. In ideal disk drive system, the tracks of the data storage disk are non perturbed circles situated about the center of the disk and track centerline is located at known constant radius from the center of the disk. Due to complexity and precision, servo tracks of disk drives are written at the time of manufacture with the equipment of servo track writer the process is called *Off-line Servo Writing or Low level formatting*. The *NRRO* disturbance during the servo track writing process causes the written servo tracks to deviate from the ideal circular shape. The offset distance between the ideal servo track and the wavy actual servo track is the *servo track writing error*. It is synchronous with the disk rotation. Therefore, it is one kind of *RRO* disturbance and has been well known as *written-in RRO* in industry. The perturbed shape of the tracks *(written-in RRO)* causes the head positioning function complicated as the servo control needs to move the head continuously to reposition during the track following to keep with the constantly changing radius of the track centerline with respect to the center of the spinning disk. The *written-in RRO* induces the unwanted head motion because the servo system attempts to make the position error zero. The unwanted head motion encroaches on the adjacent tracks i.e. inner track path gets interfered with the outer track path and hence may cause the customers data to be lost and hence not recoverable data error. Therefore, estimation and compensation of servo track writing error is very essential in enhancing the reliability of high track density disk drives.

In conventional systems, the *STW* (Servo Track Writer) is used to measure the *written-in RRO* for each track directly and generate the compensation values for each track to position the head at the center of the ideal track such that the head trace the ideal servo track. In the above case the servo track writer has to measure the *written-in RRO* of each track of disk one track at a time. Present disk drives have two or more disks with a track density of 168000 TPI, rotation speed of 7200 revolutions per minute(RPM), therefore the STW has to be busy for several hours to generate the compensation values for each track for one disk drive. Servo track writers are very expensive and there are very few servo track writers at the manufacturing place. Thus, if the servo track writers are tied up for several hours for one disk drive, the manufacturing throughput and efficiency are decreased. Therefore, it will be advantageous if there was a system which can generate *written-in RRO* values for the compensation of *written-in RRO* with out using STW which can lead to increase in manufacturing throughput. The compensation of the servo track writing error with out the use of STW has been widely used in industry. A non-casual filter to generate the compensation values was proposed (Ho *et al.,* 2000, 2001) in which non-casual time-domain impulse response was measured and it was convolved with *position error signal* (PES) to estimate the *servo track writing error*. The compensation values are called *embedded runout correction* values (ERC) which are stored with in the servo portions of each servo sector in a disk. During the track following operation when ever the head passes over the servo sector it reads the previously stored *ERC* compensation values and produces the compensation signal which is fed to the track following compensator which helps the head to follow the ideal track. However, the non-casual impulse response obtained by above description is not accurate as the modeling of the transfer function is done in non real time, and it is impractical to develop the non-casual impulse response of the inverse function for an individual disk drive. The method of measuring the impulse response more quickly than others was proposed (Melrose *et al*., 2003) where drives actual response was used in real time. The inverse impulse response was calculated by introducing an impulse to the head servo system and measuring the system response in time domain. The impulse may be written to ERC field of servo track or injected into PES signal derived from the servo sector. The response is transformed into frequency domain to form error transfer function, reciprocal is taken, transformed into time domain to form inverse impulse response which is convolved with the *position error signal* (PES) to generate the compensation values. The compensation values for all tracks can be obtained by convolving the PES values with impulse response obtained from writing impulse inputs to a single track. Accordingly, if the impulse response is obtained by writing impulse inputs to a single track it is advantageous to choose the track which is located outwards from the center of the disk as the outer tracks are more effected than inner tracks due to *written-in RRO*. In (Morris *et al.,* 2000), the transfer function of the servo system was measured at selected frequencies that are integral multiples of disk rotation because *RRO* component of *position error signal* (PES) only has frequency components that are integral multiples of spindle frequency. This occurs because the *RRO* is only measured at servo fields and servo fields are equally spaced around the disk. Transfer function is measured at discrete harmonics between the spindle frequency and one-half the number of the servo fields multiplied by the spindle frequency to obtain the time-domain sequence of runout values. Frequency-domain impulse response was measured and it was divided by the Fast Fourier Transform of *PES* which results in sequence of frequency-domain compensation values. The inverse Fast Fourier Transform of frequency-domain compensation values produces a sequence of time-domain compensation values which are used as the compensation signal in the servo sector which the controller uses to align head to ideal track. However, the system parameters effect the estimation of the servo track writing error. The initial conditions of the plant (position, velocity, bias torque) affect the system performance. Especially, torque disturbances such as flex cable bias and windage may cause the estimate of *servo track writing error* to diverge fast because *voice coil motor* (VCM) actuator of disk drives is marginally stable system (poles are nearly on unit circle). Thus, the head does not trace the ideal track instead touches the adjacent tracks and hence causes serious error during data writing operation. Nonetheless, the effect of torque disturbance on the estimate of servo track writing error was not considered in the previous algorithms (Ho *et al.,* 2000, 2001; Morris *et al.,* 2000; Melrose *et al.,* 2003).

In this study, we present a new method of compensating for the servo track writing error. In our study we consider the effect of the system parameters on the estimation of the *servo track writing error*. For the purpose, we introduce the state predictor that uses *PES* and controller output signal for its functionality. It can be shown that the predictor error has a lot of information about the *servo track writing error*. Hence, the predictor error is processed to produce the estimate of *servo track writing error*. As mentioned, the torque disturbances may cause the estimate of *servo track writing error* to diverge. In this study, the proper selection of the predictor gain can minimize the effect of disturbance on the estimate of *servo track writing error*. The estimate of *servo track writing error* obtained by processing the predictor error contains *NRRO* disturbance as well as *RRO* disturbance. The comb filter is used which extracts the signals synchronized with disk rotation to eliminate the *NRRO* component from the estimate of *servo track writing error*. Finally, we present the simulation results to demonstrate the effectiveness of our correction algorithm of *servo track writing error*.

II. HEAD-POSITIONING SERVO CONTROL SYSTEM

1. Servo Track Writing Error

A disk drive is a data storage device which stores digital data in concentric circles or tracks on the surface of a data storage disk, sputtered with a thin magnetic layer or recording medium. Data is read from or written to a track with the help of giant magneto-resistive (GMR) head which converts digital data into magnetic pulses and vice versa. The head floats several micro inches over the disk surface while the disk spins about its center at a constant angular velocity. The three main control modes of the head-positioning servo system are track seek mode, track settle mode and track-following mode (Fan *et al.*, 1995). The track seek is to move head from present track to target track as quickly as possible, track settle is to guide the head safely and precisely to the center of the target track, and the track-following is to keep the head positioned over the center of target track as precisely as possible in the presence of external disturbances. Current hard disk drives uses a combination of classical control techniques, such as proximate time optimal control technique (PTOS) in seek stage, and lead-lag compensators, PID controllers in case of track following stage, plus notch filters to remove the effect of high frequency resonance modes (see, e.g., Franklin, Powell,& Work-man, 1998; Fujimoto, Hori, Yamaguchi, & Nakagawa, 2000; Goh, Li, Chen, Lee, &Huang, 2001; Gu & Tomizuka, 2000; Hara, Hara, Yi, & Tomizuka, 1999;Huang, Messener, &Steele, 1997;

Ishikawa & Tomizuka, 1998; Iwashiro. Yatsu, & Suzuki, 1999; Yamaguchi, Soyama, Hosokawa, Tsuneta, & Hirai, 1996). The position information needed for the head-positioning control is written on the disk with the servo track writer (STW) at the time of manufacture which is called *Off-line Servo Writing or Low level formatting.* Disk is divided into large number of concentric servo tracks and each track has several servo sectors equally spaced along the track. Each servo sector contains track identification field and a group of servo bursts (small magnetic transitions) which head reads to determine the head position in reference to track center. An example of the four servo bursts (A, B, C, D) written in a particular servo sector is depicted in Fig. 1.

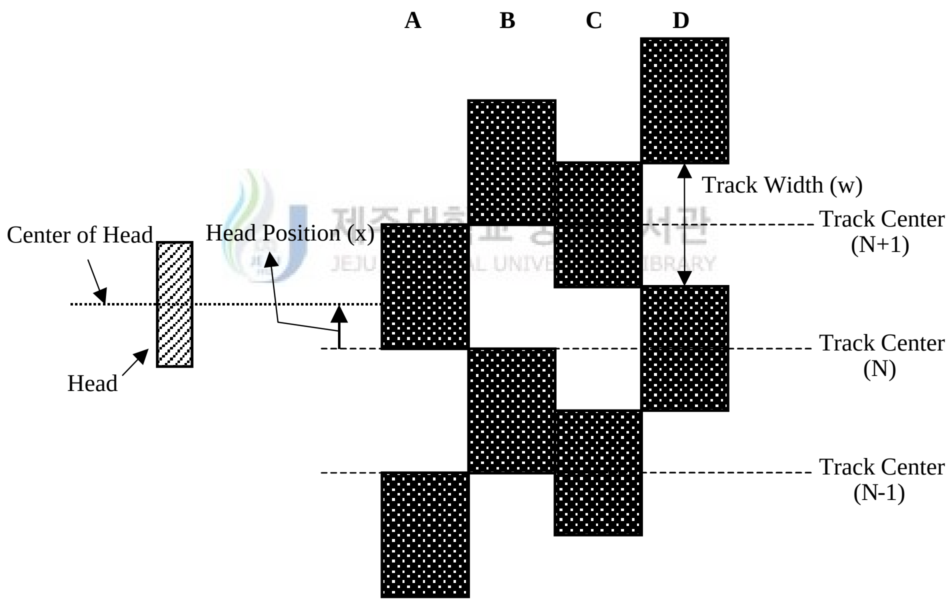
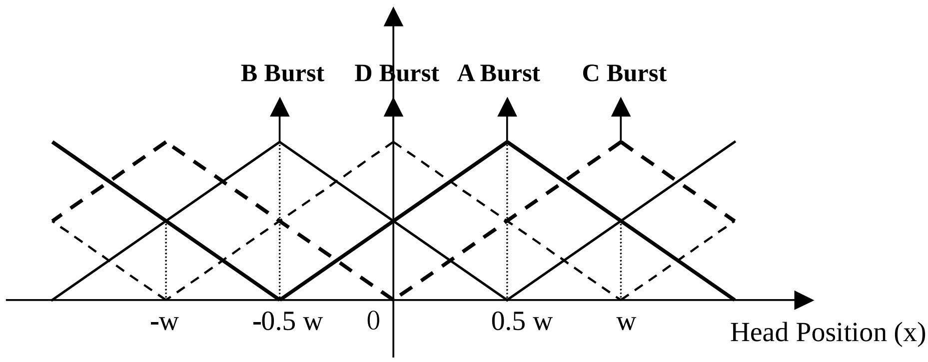
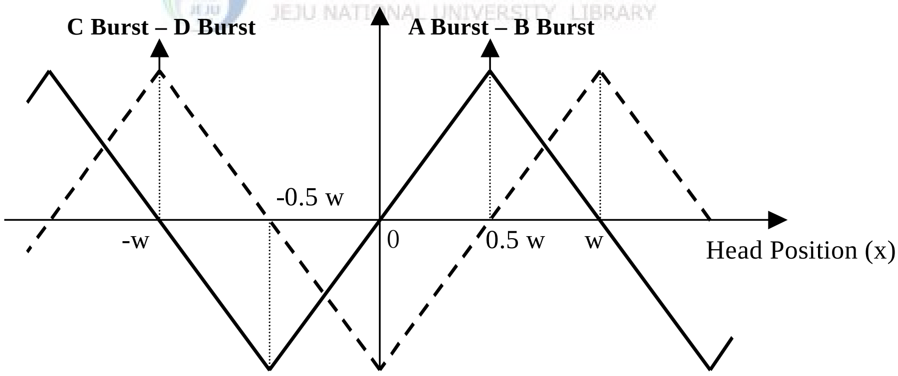


Fig. 1. Arrangement of four servo bursts in a servo sector.

The amplitude of readback signal from the servo burst is proportional to the fractional width of head over the burst. The amplitude of readback signal varying with the head position is depicted in Fig. 2(a). The position information needed for the track following servo is the difference of A and B bursts called primary *PES* depicted in Fig. 2(b). In practice, the primary *PES* exhibits nonlinear characteristics near the track boundary so, quadrature type *PES* obtained by the difference of C and D bursts is additionally used to determine the position of the head near the track boundary.



(a) Amplitude of readback signal from burst.



(b) Primary and quadrature PES.

Fig. 2. PES versus relative head position to track center.

As already mentioned, when the servo tracks being written by the *servo track writer,* the *servo track writing error* can be induced by various mechanical imperfections. In an ideal disk drive system, servo tracks are concentric circles situated about the center of the disk i.e. the servo tracks center line are located at constant radius from center of the disk. In actual system, however, it is difficult to write servo tracks as perfect circles as shown in Fig. 3.

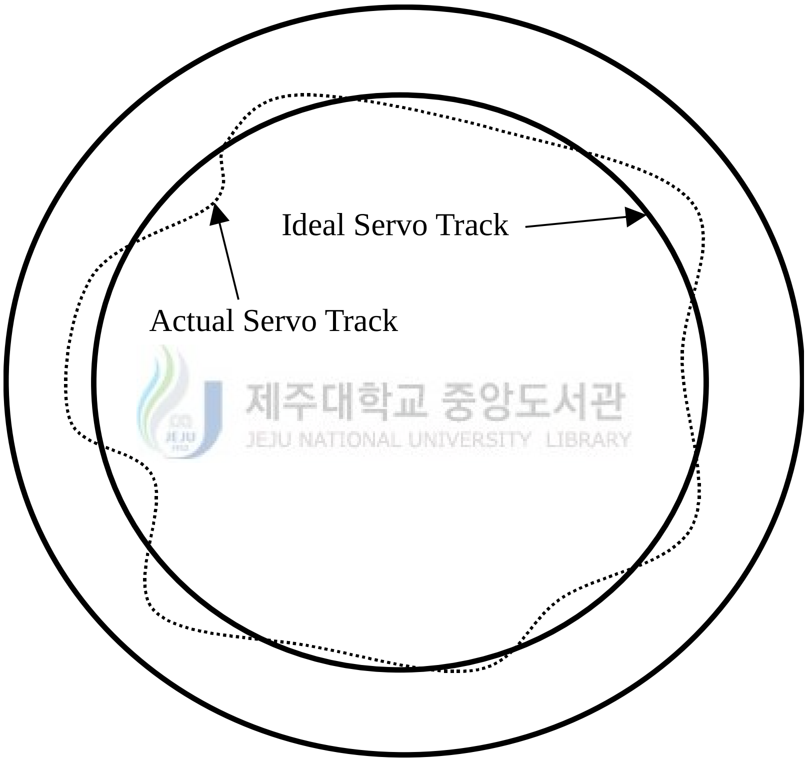


Fig. 3. Servo track writing error.

Track eccentricity can be introduced because the head to create the servo tracks cannot follow exactly the ideal path due to mechanical imperfections such as disk vibration, spindle vibrations, and external vibrations. The offset distance between the ideal servo track and the eccentric actual servo track is called the *servo track writing error.* Moreover, *servo track writing error* varies from track to track. The error can be treated as one kind of *RRO* disturbance because it is synchronous with the disk rotation. It has been well known as *written-in RRO* in industry.

As the track densities increase the *written-in RRO* began to limit the track pitch. Specifically, the waviness of actual track can result in the encroachment of adjacent tracks i.e. the inner track path can be interfered with outer track path and hence leads to data loss and irrecoverable errors during data writing operation. This occurs, because the servo control tries to make the position error caused due to servo track writing error zero so that the head trace the actual track instead of the wavy real track. This causes unwanted motion of head which leads to interference with the adjacent tracks. Especially, this is severe in case of high track density disk drives. Therefore, to avoid limitations on the track pitch and for the reliable operation of the disk drive and increase the manufacturing throughput, we need to estimate the *servo track writing error* for every track and compensate for it.

2. System Model

In disk drives rotary *voice coil motor* (VCM) is generally used to move the heads on the disk surface. Neglecting the high-frequency flexible modes, we have the following dynamic model of the VCM actuator:

$$\dot{x} = v,\ \dot{v} = K_a(u + w)\,. \tag{1}$$

Here, the constant K_a represents the acceleration constant of actuator. The variables x and v represent absolute head position and head velocity, respectively. In fact, the variable x indicates the gap between the head and the ideal servo track. However, the position information that we can measure is *PES* x_{PES} which is the gap between the head and the actual servo track. The relationship between x and x_{PES} is depicted in Fig. 4.

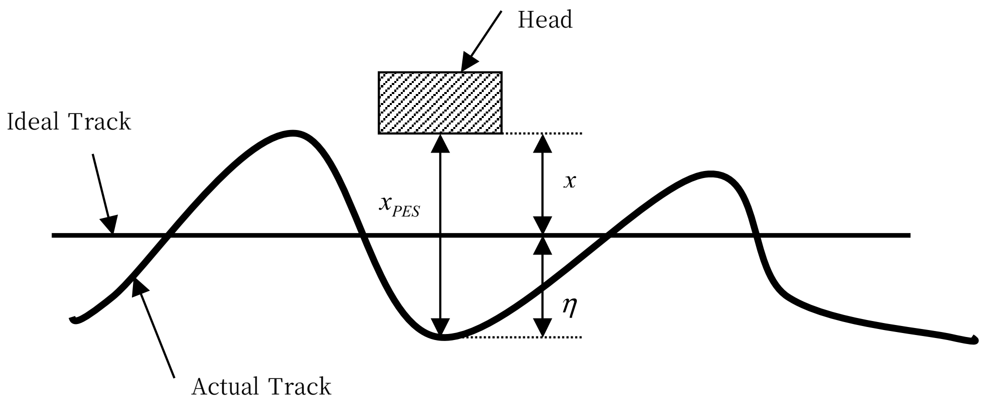


Fig. 4. Relationship between PES and head position.

From Fig. 4, we observe that

$$x_{PES} = x + \eta \,, \tag{2}$$

where η denotes the *servo track writing error*.

The control input u denotes *VCM* current and the other input w represents the disturbance torque acting on the actuator due to the flexible cable attached to the actuator. In practice, the disturbance torque varies depending on actuator positions and movement direction. However, the disturbance can be assumed to be constant with time when the actuator is almost at rest for track-following operation. Thus, we have

$$\dot{w} = 0 \,. \tag{3}$$

The above equations (1) and 2 can be expressed in standard state-space form as follows

$$\dot{\mathbf{x}} = \boldsymbol{A}\mathbf{x}(t) + \boldsymbol{B}u(t) \,. \tag{4}$$

$$x_{PES}(n) = \mathbf{C}\mathbf{x}(n) + \eta(n) \,. \tag{5}$$

Substituting the transition, measurement matrix and state vector in (4) and (5) we have

$$\begin{bmatrix} \dot{\boldsymbol{x}}(t) \\ \dot{\boldsymbol{v}}(t) \\ \dot{\boldsymbol{w}}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & \boldsymbol{K}_A \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{v}(t) \\ \boldsymbol{w}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \boldsymbol{K}_A \\ 0 \end{bmatrix} [u(t)], \tag{6}$$

$$x_{PES} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \boldsymbol{x}(t) \\ \boldsymbol{v}(t) \\ \boldsymbol{w}(t) \end{bmatrix} + \eta(t). \tag{7}$$

The PES x_{PES} is obtained at discrete times by taking the periodic samples of readback signal whenever head passes over the servo sectors. Therefore, we need to develop the discrete equivalent to the continuous-time system given by (6) and (7). Since the control input is kept constant throughout the sample period by digital to analog (D/A) converter, the zero order hold method can be used for the development.

If we define

$$\begin{aligned} &\boldsymbol{A_d} = e^{\boldsymbol{A}\boldsymbol{T_S}}, \qquad \boldsymbol{B_d} = \int_0^{\boldsymbol{T_S}} e^{\boldsymbol{A}\eta} d\eta \boldsymbol{B}, \\ &\boldsymbol{C_d} = \boldsymbol{C}, \end{aligned} \tag{8}$$

where $\boldsymbol{T_s}$ is the sampling time.

The discrete state model from the continuous-time models with out delay can be written in standard form as

$$\dot{\mathrm{x}} = \boldsymbol{A_d}\mathbf{x}(t) + \boldsymbol{B_d}u(t),$$

$$x_{PES}(n) = \mathbf{C}_d\mathbf{x}(n) + \eta(n), \tag{9}$$

$$\begin{bmatrix} \boldsymbol{x(n+1)} \\ \boldsymbol{v(n+1)} \\ \boldsymbol{w(n+1)} \end{bmatrix} = \begin{bmatrix} 1 & \boldsymbol{T_s} & \boldsymbol{K_a T_s^2}/2 \\ 0 & 1 & \boldsymbol{K_a T_s} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \boldsymbol{x(n)} \\ \boldsymbol{v(n)} \\ \boldsymbol{w(n)} \end{bmatrix} + \begin{bmatrix} \boldsymbol{K_a (T_s - T_d)^2}/2 \\ \boldsymbol{K_a (T_s - T_d)} \\ 0 \end{bmatrix} u(n). \tag{10}$$

Considering the effect of control delay by microprocessor execution additionally, we can have the following discrete-time model of actuator:

$$\begin{aligned} \mathbf{x}(n+1) &= \mathbf{A}\mathbf{x}(n) + \mathbf{B}_0 u(n) + \mathbf{B}_1 u(n-1), \\ x_{PES}(n) &= \mathbf{C}\mathbf{x}(n) + \eta(n). \end{aligned} \tag{11}$$

Here, state vector $\mathbf{x}$, system matrix $\mathbf{A}$, column vectors $\mathbf{B}_0$, $\mathbf{B}_1$ and row vector $\mathbf{C}$ are defined by

$$\begin{aligned} &\mathbf{x} = \begin{bmatrix} x \\ v \\ w \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 1 & T_s & K_a T_s^2/2 \\ 0 & 1 & K_a T_s \\ 0 & 0 & 1 \end{bmatrix}, \\ &\mathbf{B}_0 = \begin{bmatrix} K_a (T_s - T_d)^2/2 \\ K_a (T_s - T_d) \\ 0 \end{bmatrix}, \quad \mathbf{B}_1 = \begin{bmatrix} K_a (2T_s - T_d) T_d/2 \\ K_a T_d \\ 0 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^T. \end{aligned} \tag{12}$$

Here, T is the transpose of the matrix, the constants T_s and T_d are sampling period and control delay, respectively.

The matrices $\mathbf{A}$, $\mathbf{B}_0$, $\mathbf{B}_1$, $\mathbf{C}$ can be determined as follows

$$\boldsymbol{A} = e^{\boldsymbol{A}T_s}, \qquad \boldsymbol{B_0} = \int_{0}^{mT_s} e^{\boldsymbol{A}\eta}\boldsymbol{B}d\eta,$$
$$\boldsymbol{C_d} = \boldsymbol{C}, \qquad \boldsymbol{B_1} = \int_{mT_s}^{T_s} e^{\boldsymbol{A}\eta}\boldsymbol{B}d\eta. \tag{13}$$

In order to put equation (11) in standard state-space form, we must eliminate the term $u(n-1)$. To do this we augment the state vector as

$$\mathbf{x}_p(n) = \left[x(n) \quad v(n) \quad w(n) \quad u(n-1)\right]^T. \tag{14}$$

Then, the resulting state equation becomes

$$\begin{aligned} \mathbf{x}_p(n+1) &= \mathbf{A}_p\mathbf{x}_p(n) + \mathbf{B}_p u(n), \\ x_{PES}(n) &= \mathbf{C}_p\mathbf{x}_p(n) + \eta(n). \end{aligned} \tag{15}$$

Here $\mathbf{A}_p$, $\mathbf{B}_p$, and $\mathbf{C}_p$ are defined as follows:

$$\mathbf{A}_p = \begin{bmatrix} \mathbf{A} & \mathbf{B}_1 \\ 0 & 0 \end{bmatrix}, \quad \mathbf{B}_p = \begin{bmatrix} \mathbf{B}_0 \\ 1 \end{bmatrix}, \quad \mathbf{C}_p = \begin{bmatrix} \mathbf{C} & 0 \end{bmatrix}. \tag{16}$$

3. Design of Baseline Controller

The object of servo control is to make head trace the center of the ideal servo track precisely. To achieve this servo control uses state feedback controller and state estimator. The control law is simply the feedback of a linear combination of all state elements. The state feedback controller can be written as

$$u(n) = -\mathbf{K}_f \hat{\mathbf{x}}_p(n) = -K_x x(n) - K_v \hat{v}(n) - w(n)\,. \tag{17}$$

Here, $\hat{\mathbf{x}}_p(n)$ is state estimate vector defined by

$$\hat{\mathbf{x}}_p(n) = \begin{bmatrix} x(n) & \hat{v}(n) & w(n) & u(n-1) \end{bmatrix}^T, \tag{18}$$

and $\mathbf{K}_f$ is feedback gain vector defined by

$$\mathbf{K}_f = \begin{bmatrix} K_x & K_v & 1 & 0 \end{bmatrix}, \tag{19}$$

where K_x and K_v are the position gain and velocity gain, respectively. Note that the estimate $\hat{w}$ is included in feedback to suppress the effect of constant disturbance torque w and the previous control input $u(n-1)$ is excluded in feedback.

State estimator is used to estimate entire state vector from the position measurement and control input. The current state estimator is introduced, which estimates based on the current measurement and hence provides fastest response to unknown disturbances.

$$\bar{\mathbf{x}}_p(n+1) = \mathbf{A}_p\hat{\mathbf{x}}_p(n) + \mathbf{B}_p u(n)\,, \tag{20}$$

$$\hat{\mathbf{x}}_p(n) = \bar{\mathbf{x}}_p(n) + \mathbf{L}_e[x_{PES}(n) - \mathbf{C}_p\bar{\mathbf{x}}_p(n)]. \tag{21}$$

Here $\bar{\mathbf{x}}_p$ is the prediction estimate vector and $\mathbf{L}_e$ is estimator feedback gain vector. The combination of the control law (17) and the state estimator (20) and (21) allows us to obtain the transfer function from *PES* x_{PES} to control input u. Substituting (17) in (20), we have

$$\bar{\mathbf{x}}_p(n+1) = (\mathbf{A}_p - \mathbf{B}_p\mathbf{K}_f)\hat{\mathbf{x}}_p(n)\,. \tag{22}$$

Combining (17), (21), and (22) together yields the following equation describing the dynamics of the combined controller:

$$\begin{aligned} \bar{\mathbf{x}}_p(n+1) &= \mathbf{A}_f(\mathbf{I} - \mathbf{L}_e\mathbf{C}_p)\bar{\mathbf{x}}_p(n) + \mathbf{A}_f\mathbf{L}_e x_{PES}(n), \\ u(n) &= -\mathbf{K}_f(\mathbf{I} - \mathbf{L}_e\mathbf{C}_p)\bar{\mathbf{x}}_p(n) - \mathbf{K}_f\mathbf{L}_e x_{PES}(n), \end{aligned} \tag{23}$$

Here $\mathbf{I}$ is identity matrix and $\mathbf{A}_f = \mathbf{A}_p - \mathbf{B}_p\mathbf{K}_f$. From (2), (15), (23), we see that the effect of *servo track writing error* η on the control system can be described as

$$\mathrm{x(z)} = \frac{-P(\mathrm{z})C(\mathrm{z})}{1+P(\mathrm{z})C(\mathrm{z})}\eta(z), \tag{24}$$

where $P(z)$ is the transfer function of the actuator and $C(z)$ is the transfer function of the combined controller. That is,

$$\begin{aligned} P(z) &= \mathbf{C}_p(z\mathbf{I}-\mathbf{A}_p)^{-1}\mathbf{B}_p, \\ C(z) &= \mathbf{K}_f\mathbf{L}_e + \mathbf{K}_f(\mathbf{I}-\mathbf{L}_e\mathbf{C}_p)[z\mathbf{I}-\mathbf{A}_f(\mathbf{I}-\mathbf{L}_e\mathbf{C}_p)]^{-1}\mathbf{A}_f\mathbf{L}_e. \end{aligned} \tag{25}$$

Since *servo track writing error* η is periodic function, we can see that the actual head position x is also periodic function in the steady-state. This implies that the actual head position x does not converge to zero and fluctuates periodically due to *servo track writing error.* The unwanted fluctuations can cause the head to write on the adjacent tracks, which in turn produces irrecoverable operation error. In the subsequent section, we present a new correction algorithm of *servo track writing error.*

III. DESIGN OF SERVO TRACK WRITING ERROR ESTIMATOR

Before designing *written-in RRO estimator*, we briefly touch on the prior work. As shown in Fig. 5, the *written-in RRO estimator* in the prior work uses *PES* and controller output to estimate the *servo track writing error*. The controller output passes through the model plant $\hat{P}(z)$ to obtain the absolute head position which is subtracted from x_{PES} to give the estimate of *written-in RRO* $\hat{\eta}$. In Fig. 5, initial state $\mathbf{x}_p(0)$ contains initial head position $x(0)$, velocity $v(0)$, and constant bias torque $w(0)$.

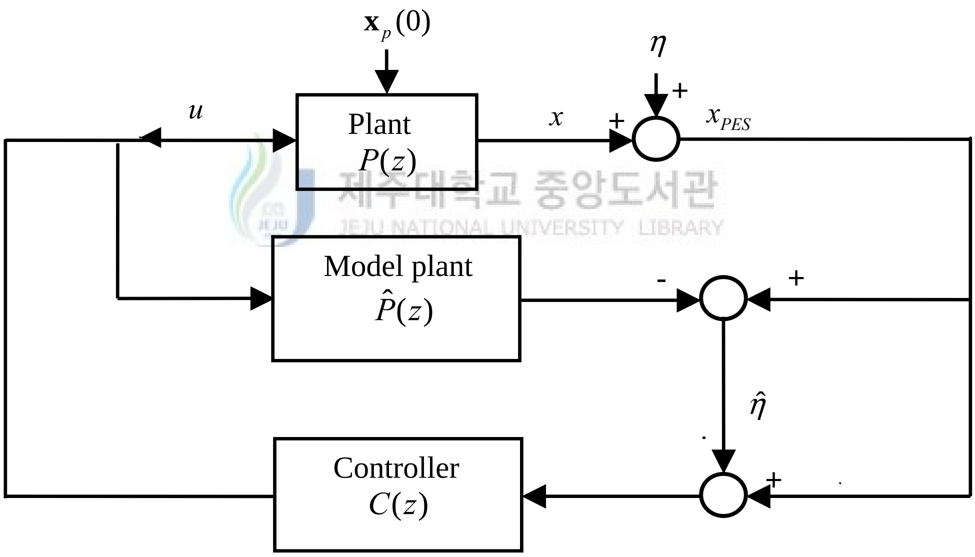


Fig. 5. Written-In RRO estimator in prior work.

If the model plant is exact, the estimate can be written as

$$
\begin{aligned}
\hat{\eta}(z) &= x(z) + \eta(z) - \hat{P}(z)u(z), \\
&= \mathbf{C}_p(z\mathbf{I} - \mathbf{A}_p)^{-1}\mathbf{x}_p(0) + P(z)u(z) + \eta(z) - \hat{P}(z)u(z), \qquad (26) \\
&= \mathbf{C}_p(z\mathbf{I} - \mathbf{A}_p)^{-1}\mathbf{x}_p(0) + \eta(z).
\end{aligned}
$$

The estimation error $\hat{\eta} - \eta$ will be zero if initial plant state $\mathbf{x}_p(0)$ is zero. However, the estimation error might grow in the presence of initial plant state. Especially, it diverges with the fast rate (the second power of time) in the presence of constant bias torque. To see this, observe from (12) and (16) that

$$\det(z\mathbf{I} - \mathbf{A}_p) = z(z-1)^3, \qquad (27)$$

$$\mathbf{C}_p(z\mathbf{I} - \mathbf{A}_p)^{-1}\mathbf{x}_p(0) = \frac{1}{z-1}x(0) + \frac{1}{(z-1)^2}v(0) + \frac{1}{(z-1)^3}w(0). \qquad (28)$$

Now, we are ready to present a new *written-in RRO estimator* whose error decays to zero even in the presence of constant bias torque. Diagram of the proposed *written-in RRO estimator* is shown in Fig. 6.
The *written-in RRO estimator* is described by

$$\bar{\mathbf{x}}_c(n+1) = \mathbf{A}_p\hat{\mathbf{x}}_c(n) + \mathbf{B}_p u(n), \qquad (29)$$

$$\hat{\mathbf{x}}_c(n) = \bar{\mathbf{x}}_c(n) + \mathbf{L}_c\left(x_{PES}(n) - \mathbf{C}_p\bar{\mathbf{x}}_c(n)\right), \qquad (30)$$

$$\hat{\eta}(n) = K_c\left(x_{PES}(n) - \mathbf{C}_p\bar{\mathbf{x}}_c(n)\right). \qquad (31)$$

This is similar to the state estimator of baseline controller in (20) and (21). The estimate of *written-in RRO* is the product of some constant K_c and prediction error. The value of the constant K_c will be determined shortly.

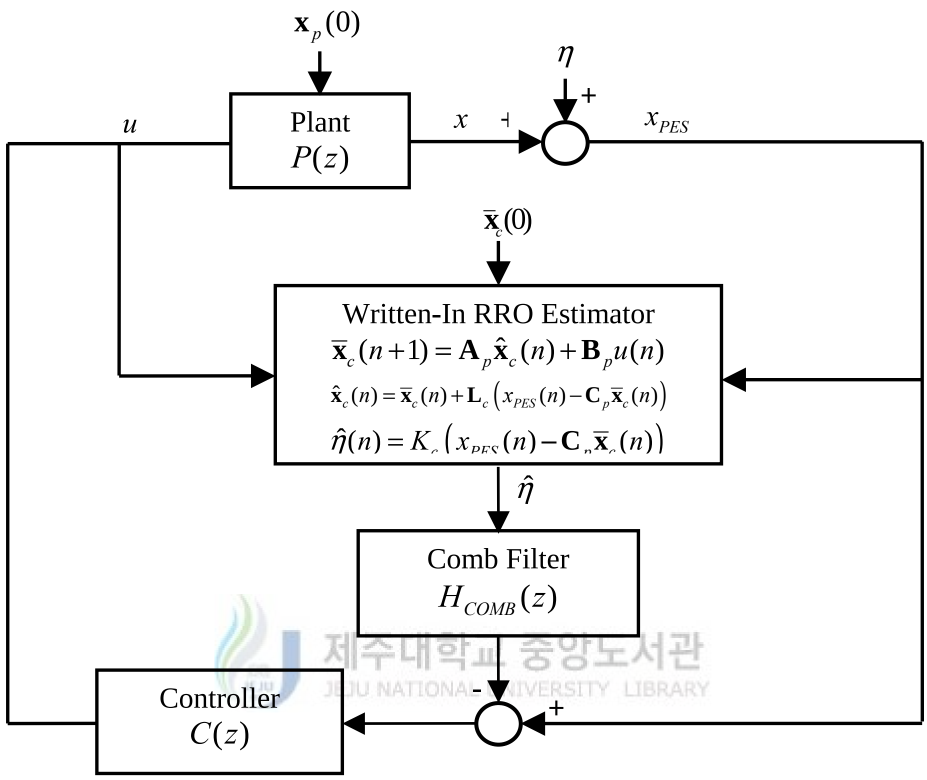


Fig. 6. Proposed written-in RRO estimator.

Using (15), we can rewrite (30) as

$$\hat{\mathbf{x}}_c(n) = \bar{\mathbf{x}}_c(n) + \mathbf{L}_c\mathbf{C}_p\left(\mathbf{x}_p(n) - \bar{\mathbf{x}}_c(n)\right) + \mathbf{L}_c\eta(n), \tag{32}$$

and hence rewrite (29) as

$$\overline{\mathbf{x}}_c(n+1) = \mathbf{A}_p\overline{\mathbf{x}}_c(n) + \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\left(\mathbf{x}_p(n) - \overline{\mathbf{x}}_c(n)\right) + \mathbf{A}_p\mathbf{L}_c\eta(n) + \mathbf{B}_p u(n). \qquad (33)$$

Subtracting (33) from (15) and defining the prediction error as $\tilde{\mathbf{x}}(n) = \mathbf{x}_p(n) - \overline{\mathbf{x}}_c(n)$, we obtain

$$\begin{aligned} \tilde{\mathbf{x}}(n+1) &= \mathbf{x}_p(n+1) - \overline{\mathbf{x}}_c(n+1), \\ &= \mathbf{A}_p\left(\mathbf{x}_p(n) - \overline{\mathbf{x}}_c(n)\right) - \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\left(\mathbf{x}_p(n) - \overline{\mathbf{x}}_c(n)\right) - \mathbf{A}_p\mathbf{L}_c\eta(n), \qquad (34) \\ &= \left(\mathbf{A}_p - \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)\tilde{\mathbf{x}}(n) - \mathbf{A}_p\mathbf{L}_c\eta(n), \quad \tilde{\mathbf{x}}(0) = \mathbf{x}_p(0) - \overline{\mathbf{x}}_c(0). \end{aligned}$$

Using (15) and (34), we can describe the *written-in RRO estimator* (29-31) in standard form as

$$\begin{aligned} \tilde{\mathbf{x}}(n+1) &= \left(\mathbf{A}_p - \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)\tilde{\mathbf{x}}(n) - \mathbf{A}_p\mathbf{L}_c\eta(n), \\ \hat{\eta}(n) &= K_c\mathbf{C}_p\tilde{\mathbf{x}}(n) + K_c\eta(n). \end{aligned} \qquad (35)$$

Taking z-transform of (35), we have

$$\begin{aligned} \hat{\eta}(z) = {} & K_c\mathbf{C}_p\left(z\mathbf{I} - \mathbf{A}_p + \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)^{-1}\tilde{\mathbf{x}}(0) + \\ & K_c\left(1 - \mathbf{C}_p\left(z\mathbf{I} - \mathbf{A}_p + \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)^{-1}\mathbf{A}_p\mathbf{L}_c\right)\eta(z). \end{aligned} \qquad (36)$$

The first term on the right-hand side of (36) is the effect of initial prediction error $\tilde{\mathbf{x}}(0) = \mathbf{x}_p(0) - \overline{\mathbf{x}}_c(0)$ on the estimate $\hat{\eta}$. As a matter of fact, it can be made to decay to zero exponentially by choosing $\mathbf{L}_c$ such that $\mathbf{A}_p - \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p$ is sufficiently stable. Also, we can control the convergence rate because the VCM actuator (15) is observable system and hence all eigen values of $\mathbf{A}_p - \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p$ can be assigned arbitrarily. Especially, the effect of initial prediction error $\tilde{\mathbf{x}}(0)$ on the estimate $\hat{\eta}$ can be eliminated perfectly by reading the estimated state $\hat{\mathbf{x}}_p(0)$ from the state estimator of baseline controller (20), (21) and setting the initial state $\overline{\mathbf{x}}_c(0)$ of the *written-in RRO estimator* (29), (31) to the estimated state. What remains is to examine the second term on the right-hand side of (36). To do so, we first need to show that the following identity holds:

$$1-\mathbf{C}_p\left(z\mathbf{I}-\mathbf{A}_p+\mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)^{-1}\mathbf{A}_p\mathbf{L}_c=\frac{\det\left(z\mathbf{I}-\mathbf{A}_p\right)}{\det\left(z\mathbf{I}-\mathbf{A}_p+\mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)}. \tag{37}$$

where, $\mathbf{I}$ is the identity matrix. The proof is given in Appendix. From this identity and (27), it is evident that the second term on the right-hand side of (36) is the output of certain high-pass filter with *written-in RRO* η being the input of the filter. It is reasonable to select the constant K_c so that the high-pass filter has unity gain at $z=-1$. That is,

$$K_c=\frac{\det\left(\mathbf{I}+\mathbf{A}_p-\mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)}{\det\left(\mathbf{I}+\mathbf{A}_p\right)}. \tag{38}$$

As usual, η is high-frequency signal. Therefore, the estimate of *written-in RRO* $\hat{\eta}$ (36) is very close to the actual *written-in RRO* η. The cut-off frequency of the high-pass filter depends on the selection of $\mathbf{L}_c$. Selecting $\mathbf{L}_c$ to achieve faster rejection of initial prediction error might yield higher cut-off frequency and thus larger estimation error. The trade-off between rejection rate of initial prediction error and cut-off frequency of high-pass filter should be taken into account in the selection of $\mathbf{L}_c$.

The estimate $\hat{\eta}$ is used to correct *PES* in a feed-forward manner

$$x_{PES}^{c}(n) = x_{PES}(n) - \hat{\eta}(n). \tag{39}$$

The corrected *PES* x_{PES}^{c} is fedback to control loop. Due to that, the control loop does not attempt to follow the actual wavy track but ideal circular track. The *written-in RRO estimator* we have discussed until now should not estimate *NRRO* disturbance because control loop should track it. However, the estimate of *written-in RRO* $\hat{\eta}$ (36) contains both *NRRO* and *written-in RRO*. Now comb filter is introduced to remove *NRRO* from the estimate $\hat{\eta}$. The filter is designed in such a way that the frequency response is 1 at integer multiples of disk rotation frequency otherwise it is zero. The desired frequency response can be expressed as

$$H_{comb}^{*}(e^{j\omega}) = \begin{cases} 1 & \omega = \dfrac{2\pi}{M}m, \quad m = 0,\ 1,\ \cdots,\ M-1, \\ 0 & \text{otherwise,} \end{cases} \tag{40}$$

where M is the number of servo sectors per track. In this study, we consider FIR comb filter described by

$$H_{comb}(z) = \frac{1}{N}\left(1 + z^{-M} + z^{-2M} + \cdots + z^{-(N-1)M}\right), \tag{41}$$

where N is the number of disk revolutions. The action of the comb filter is to average the intermediate estimate $\hat{\eta}$ at each servo sector over all of the revolutions. To put it another way, it extracts frequency components synchronized with disk rotation and attenuates the other components resulted from *NRRO* disturbance. The accuracy of the comb filter (41) depends on the number of revolutions N. As the number of revolutions increases then the frequency response of the filter (41) approaches the desired one (40). However, every revolution increases the process time and thus should be minimized. The frequency response of the comb filter for different number of revolutions is shown in Fig. 7. In Fig. 7, frequency range is from zero (m=0) to disk rotation frequency (m=1).

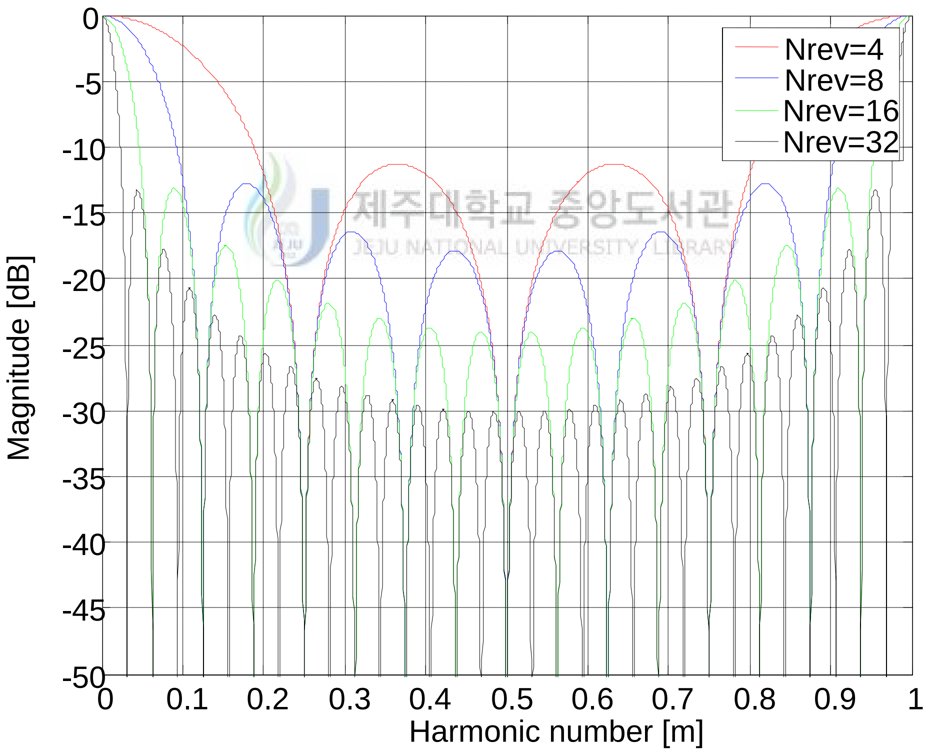


Fig. 7. Frequency response of FIR comb filter.

IV. SIMULATION RESULTS

In this section, we present some simulation results on the performance of the proposed correction algorithm discussed in the previous section. For our simulation work, we performed simulation by using *Matlab-simulink*. We used the parameters of 3.5-inch disk drive with MR head manufactured by Samsung Co., Korea. The specifications of the disk drive used are given below.

Table 1. Specifications of Hard disk drive used for simulation

Specification	Value
Storage capacity	80 GB/Platter
Track density	130,200 TPI
Track width	0.27 μm
Disk rotation speed	7,200 rpm
Servo sectors per track	232
Acceleration constant (K_a)	22,236 rad/sec^2/A
Sampling time (T_s)	35.9 μsec
Control delay (T_d)	17 μsec

First we discuss the design of state feedback controller and state estimator. The *pole-placement method* is used to determine the feedback gains of state feedback controller and state estimator. To place the poles of state feedback controller at 0.8503 ± 0.045i and state estimator at 0.432 ± 0.1169i, 0.958 we have chosen the state feedback gain and state estimator gain as

$$\mathbf{K}_f = [0.2535 \quad 1.6494 \quad 1.0000 \quad 0]. \tag{42}$$

$$\mathbf{L}_e = [0.8081 \quad 0.6552 \quad 0.1464 \quad 0]^T. \tag{43}$$

For the design of *written-in RRO estimator, pole-placement method* was used to determine the predictor gain $\mathbf{L}_c$ in (32). Generally, *written-in RRO* η is high-frequency signal. Care should be taken while choosing $\mathbf{L}_c$ because its choice to achieve the faster rejection of initial prediction error might yield larger *written-in RRO* estimation error. To place the poles of the state predictor at 0.985, 0.985, 0.985, we have chosen the predictor gain as

$$\mathbf{L}_c = [0.0443 \quad 0.0012 \quad 0 \quad 0]^T. \tag{44}$$

The simulation results are illustrated in Fig. 8 through Fig. 16. Figs. 8 to 13 show the simulation results in case of the prior work while Figs. 13 to 16 shows the simulation results in case of the proposed controller. For the emulation of *written-in RRO*, the sinusoidal signal with amplitude and frequency of 1 track and 2400 Hz (20 times of disk rotation frequency) was generated and introduced artificially into the system. Initial conditions of the plant are assumed to be one track away from the target track, 0.0012 m/sec, and 0 amp for position, velocity and previous control input, respectively. Bias torque of 0.756 N-m is assumed. The initial condition of the plant affects the system performance in case of prior work. Incase of prior work the *written-in RRO* is estimated using *PES* and control output. The *written-in RRO* estimation error becomes zero in the absence of initial plant state. However, the estimation error grows in the presence of initial plant state, which can be seen from (26). From Figs. 8, 9, and 10 we can see the effect of initial plant state on the estimation error. Estimation error remains to be some constant value due to initial head position, increases linearly with time in case of initial velocity, increases with second power of time in case of bias torque which can be seen through (28). Due to that estimation error the head cannot move to the target track center and especially it goes far away in the presence of initial velocity or bias torque, which can be seen from Figs. 11, 12, and 13. Especially, it diverges very fast in case of constant bias torque which can be noticed in the Fig. 13. This means that the head touches the adjacent tracks and hence causes serious error during data writing operation.

As described in the previous section, *written-in RRO* estimation error can be made to decay to zero in the presence of initial state using our proposed *written-in RRO* estimator. This is done by reading the initial state of plant from state estimator of base line controller and setting the initial state of *written-in RRO* estimator to that estimated state and with the proper selection of predictor gain. From Fig. 14, we can see that with the application of our *written-in RRO* estimator the estimation error before going through comb filter decreases and approaches toward zero in the presence of any initial state. However, it does not converge to zero perfectly because of high-pass filter action in (37). The choice of slower poles of the predictor than 0.985 can give more exact written-in RRO estimation error but yields slower convergence rate. The output of the *written-in RRO estimator* also contains *NRRO*, FIR comb filter is used to remove NRRO from the output of *written-in RRO estimator*. The performance of comb filter depends on the number of revolutions chosen. With the increase in number of revolutions the frequency response of the comb filter (41) approaches the desired one (40) but it results in increase of process time thus should be minimized. The revolution number of comb filter needed for *NRRO* removal is chosen as $N = 8$. The estimation error after comb filter is depicted in Fig. 15. From the figure, we see that the time of 8 revolutions was taken until the complete convergence. The output of comb filter is then used to correct the *PES* in feed-forward manner. The resultant signal is used by the controller to move the head to target track. With the help of our *written-in RRO estimator*, the head is made to trace the ideal track center instead of wavy track as depicted in Fig. 16. The simulation results show that the *written-in RRO* can be significantly compensated and hence the reliability of disk drive can be greatly enhanced with the application of our new correction algorithm.

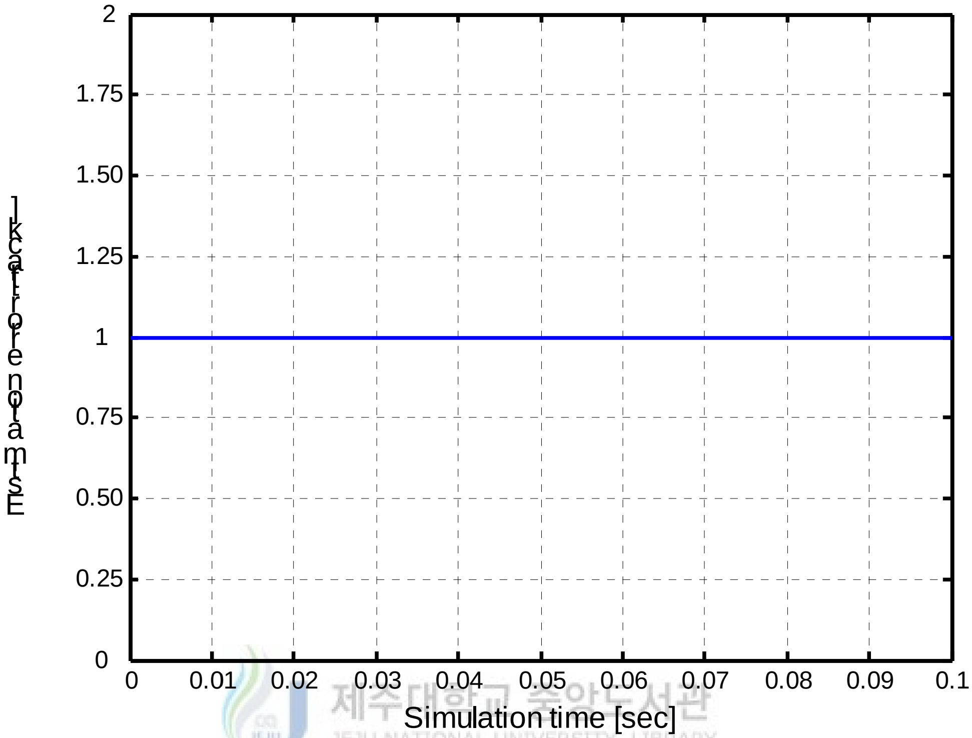


Fig. 8. Estimation error due to initial head position in case of prior work.

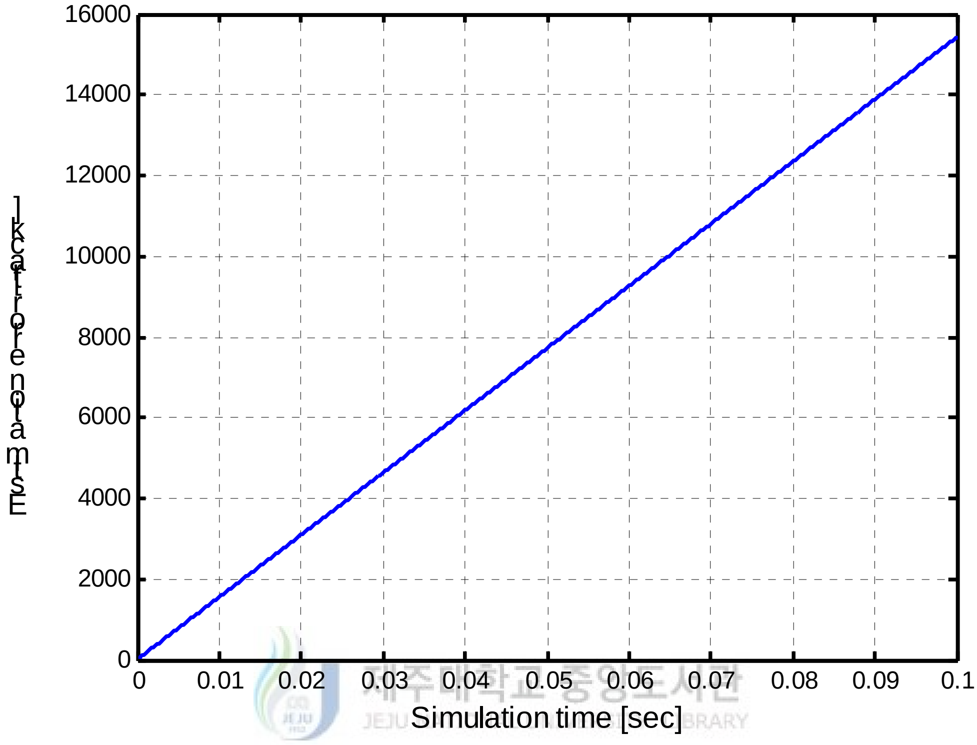


Fig. 9. Estimation error due to initial head velocity in case of prior work.

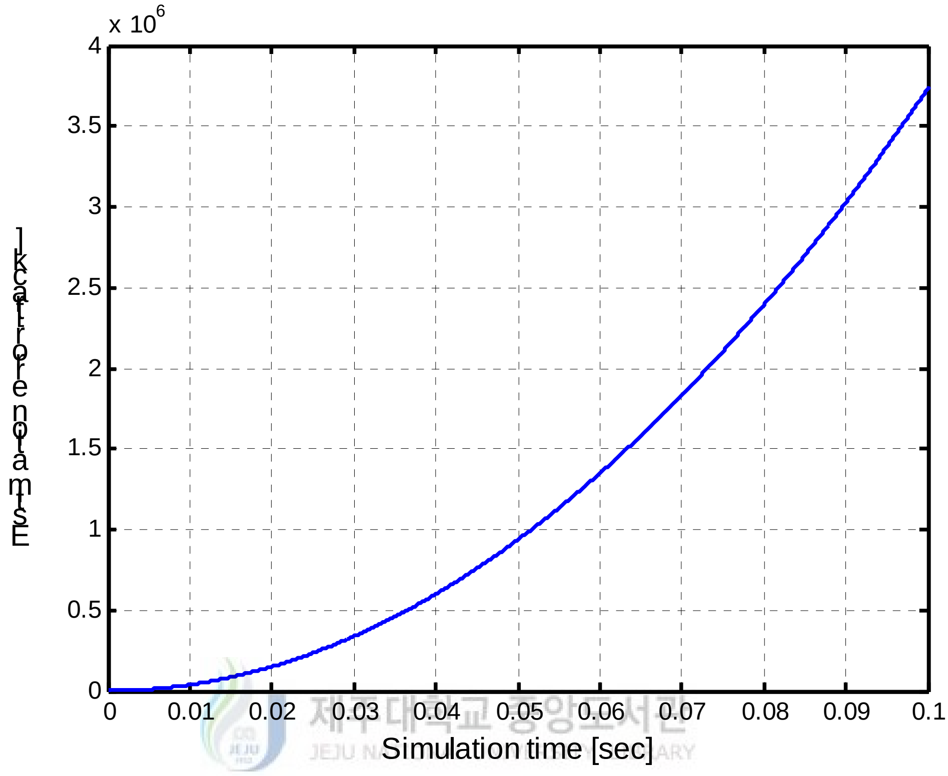


Fig. 10. Estimation error due to initial bias torque in case of prior work.

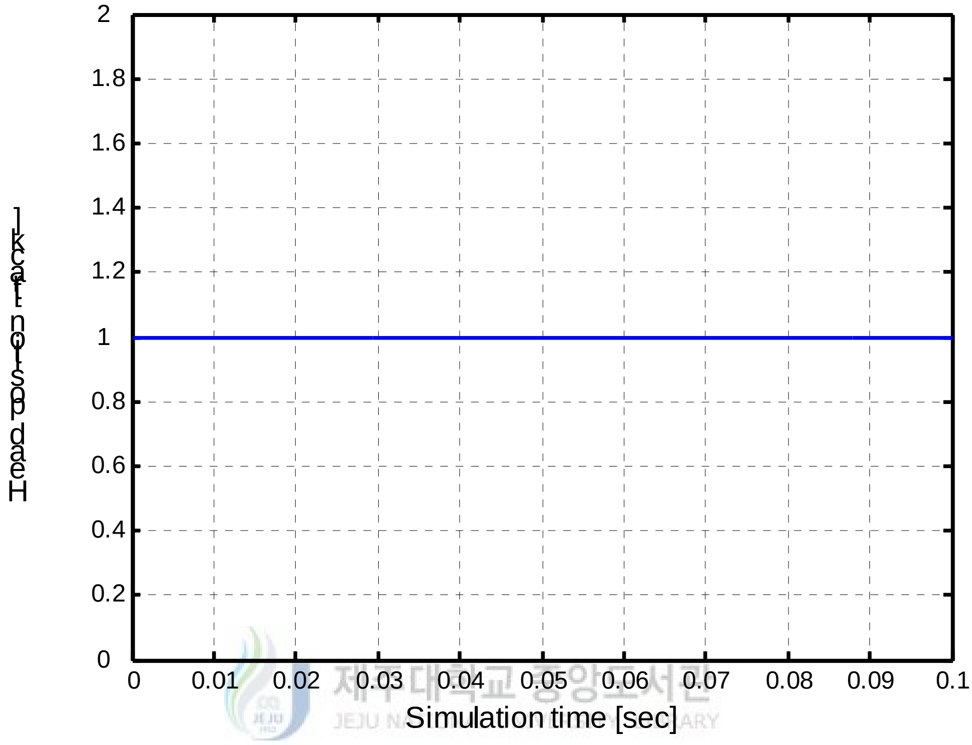


Fig. 11. Head position due to initial head position in case of prior work.

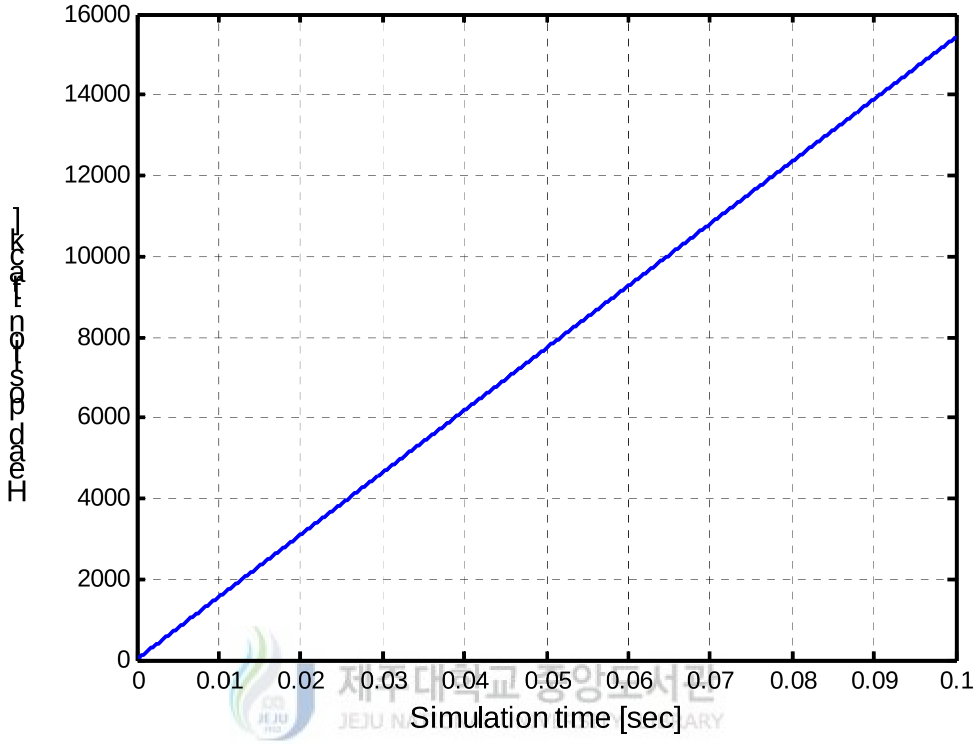


Fig. 12. Head position due to initial head velocity in case of prior work.

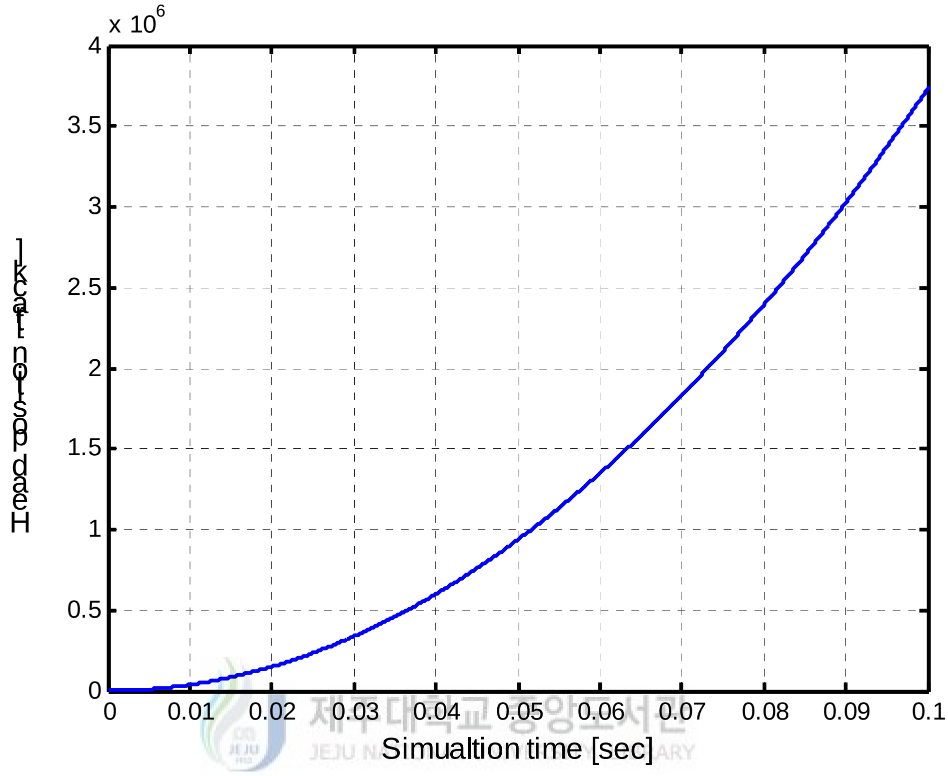


Fig. 13. Head position due to initial bias torque in case of prior work.

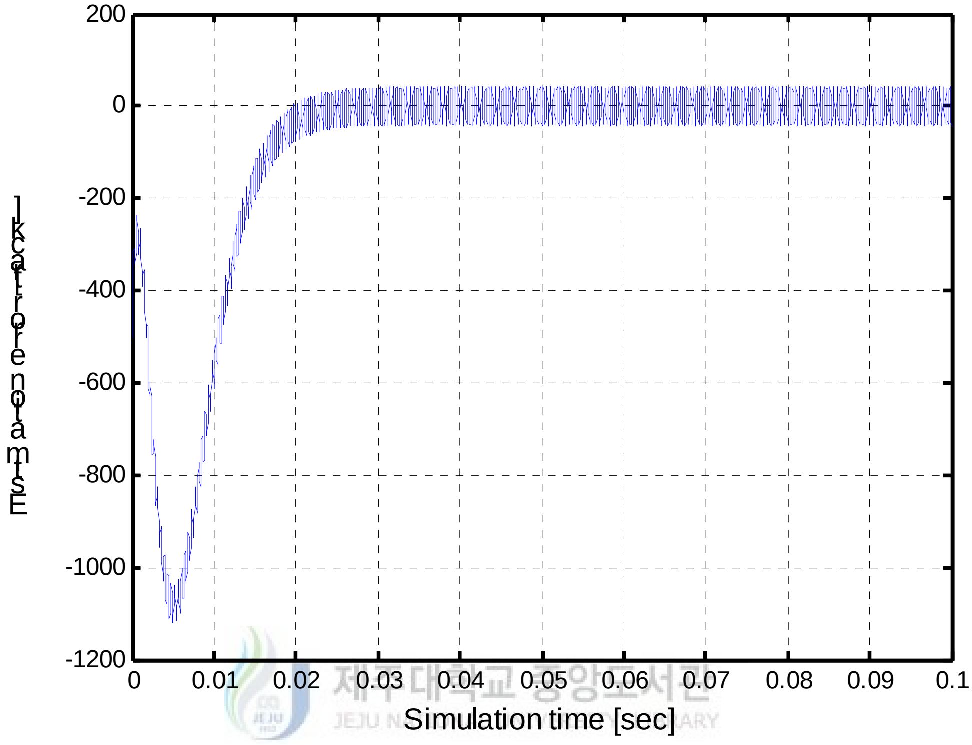


Fig. 14. Estimation error in case of proposed controller.

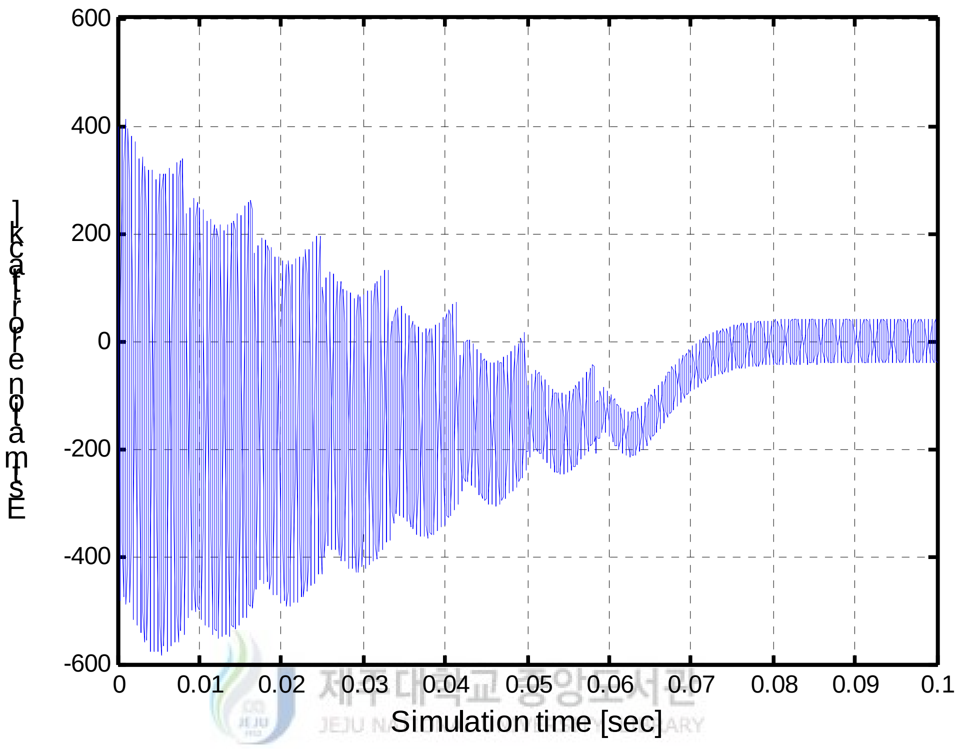


Fig. 15. Estimation error after comb filter in case of proposed controller.

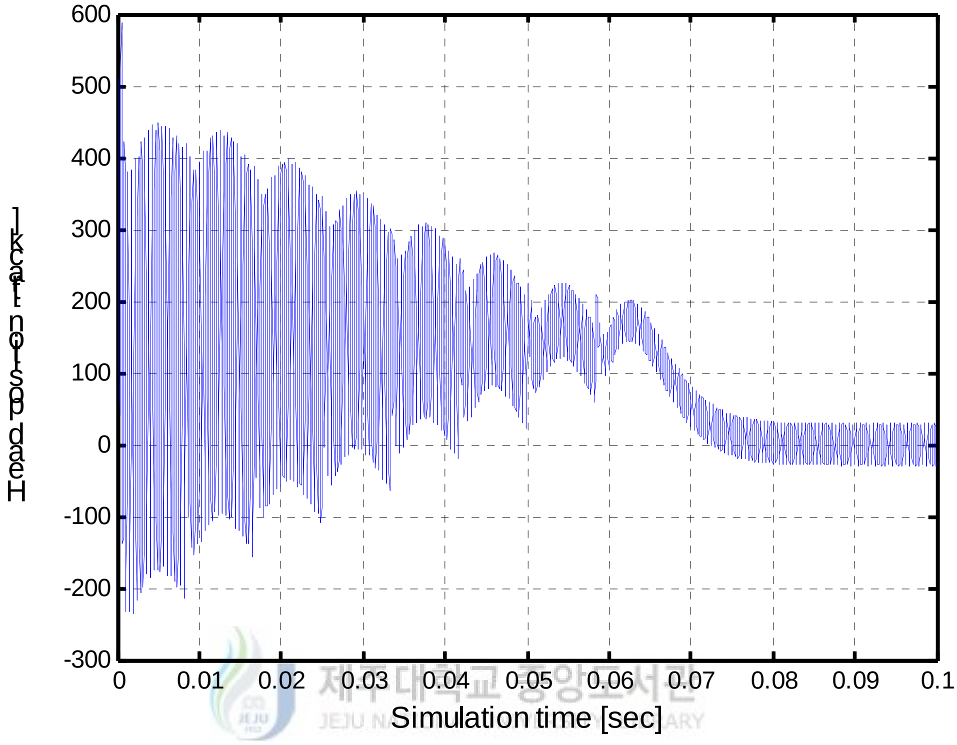


Fig. 16. Head position in case of proposed controller.

V. CONCLUSIONS

In this study, we proposed a new correction algorithm to estimate and compensate for the *servo track writing error* in disk drives. In the design of *written-in RRO estimator*, initial conditions of the plant (position, velocity, and bias torque) were considered. Due to proper selection of predictor gain and setting the initial condition of plant to the *written-in RRO estimator* the estimation error can be made decayed to zero even in the presence of flexible bias torque. The signal is compensated and the head is made to follow the centre of ideal track. The simulation results using the parameters of available hard disk through *Matlab-simulink* have demonstrated that the *written-in RRO estimator* is very effective in reducing the servo track writing error. The simulation results show that with the application of our proposed controller good track and servo performance can be obtained.

The proposed correction algorithm is robust to system parameter variations and measurement noise. Moreover, it is very effective and has fast convergence rate compared to previous algorithms. Hence this algorithm can be applied to other data storage devices such as optical drives for fast rejection of periodic disturbance.

VI. APPENDIX

< Proof of (37) >

We use the useful identity for determinants [*T. Kailath*, Prentice Hall, 1980]. If matrix $\mathbf{A}$ is $n \times m$ and matrix $\mathbf{B}$ is $m \times n$, then it holds that

$$\det\left(\mathbf{I}_m - \mathbf{BA}\right) = \det\left(\mathbf{I}_n - \mathbf{AB}\right) \tag{A1}$$

The case where $m = 1$ yields that

$$\begin{aligned}
1 - \mathbf{C}_p\left(z\mathbf{I} - \mathbf{A}_p + \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)^{-1}\mathbf{A}_p\mathbf{L}_c &= \det\left(I_n - \left(z\mathbf{I} - \mathbf{A}_p + \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)^{-1}\mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right) \\
&= \det\left(z\mathbf{I} - \mathbf{A}_p + \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)^{-1}\det\left(z\mathbf{I} - \mathbf{A}_p\right) \\
&= \frac{\det\left(z\mathbf{I} - \mathbf{A}_p\right)}{\det\left(z\mathbf{I} - \mathbf{A}_p + \mathbf{A}_p\mathbf{L}_c\mathbf{C}_p\right)}
\end{aligned}$$

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